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PHYS1001

Fundamental Mathematics

# Vector – Scalar

**Vector**

with both magnitude and direction. Here, the vector v is represented as having values v1, v2 etc in the 1st, 2nd and 3rd dimensions. From hereon, the above notation for a vector will be called the **vector component form.**

**Scalar**

A scalar is a quantity which have no direction. The scalar value for a vector is often called the **magnitude** of the vector. While we’re already familiar with the concept of magnitude as being the absolute value of a vector, we must recognise that the magnitude of a vector, expressed in its component form, can be calculated as thus:

magnitude

The Unit Vector  
A vector in the direction of the actual vector with a dimensionless length of 1.

* Thus, if we have a vector in direction x, then its unit vector will be which also points in the direction x with a length of 1.
* The relationship between the vector and unit vector is given by:  
    
   represents the magnitude of the vector . represents the components of the vector in the x, y and z planes respectively.
* The magnitude of is given by:
* Alternative perspective:  
    
   that for a vector, it is comprised of a magnitude, , and direction .

# Vector Addition/Subtraction

For and

# Dot and Cross Product

## Dot Product

### General method

is the angle between the two vectors .

* The dot product is a scalar quantity, so it has no direction.
* An example of a dot product is work, which is the dot product of force by displacement.

### Component method

If   
and

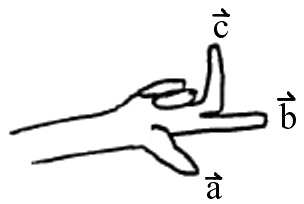
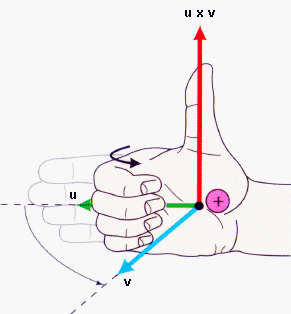
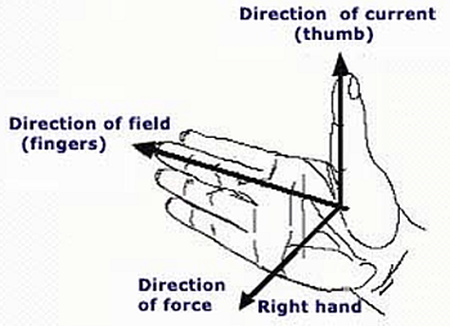
Where and describes the set of coordinates for a and b,   
  
 Then

## Cross Product

Here, c is the vector cross product of vectors a and b **in that order**. Because the vector cross product has both magnitude and direction, we determine the cross product through two steps.

### General method

Here, c is the vector cross product of vectors a and b **in that order**. Because the vector cross product has both magnitude and direction, we determine the cross product through two steps.

1. The first step is to determine the magnitude, given by:
2. The next step is to determine the direction of the vector, given by the right hand rule. The right hand rule has three variations, the right finger rule, the right curl rule and the right palm rule. These are displayed in that order below:  
     
     
   Right finger rule:  
     
   Right curl rule:  
     
     
   Note: Here, we’re finding the vector cross product of .  
     
     
     
   Right palm rule:  
     
   

### Component method

If   
and

Then the vector cross product can be determined through the determinant of a 3\*3 matrix:

Where

# Fields

The following characteristics apply to both scalar and vector fields

* Static Fields: A static field is one which is not dependent on time.
* Constant Fields: A constant field is one which assigns the same value or vector to each point in space and time.

## Scalar Field

An assignment of a number (a scalar) to each point in space and time.

x, y, z and t. This therefore makes A a scalar value, rather than a vector.

Vector Field  
An assignment of a vector to each point in space and time.

# Tricks

## Squares

* If:   
    
    
  Then:  
    
   or
* If : ,   
    
  Then:  
    
   or

Thermodynamics

Key Definitions

* Heat: The energy transferred between the boundary between a system and its environment due to a difference in temperature.
* Temperature: The mean translational kinetic energy of the particles of a system. Expressed in terms of the zeroth law, it is the variable whose value is common to two objects which are in thermal equilibrium.
* Work:
* Energy: The ability to do work.
* Entropy: A measure of the disorder of a system.

# Introduction

* Thermodynamics is the study of macroscopic amounts of matter and their thermal properties.
* There is also another branch of thermodynamics, known as **statistical mechanics**, which examines thermodynamics in terms of a large collection of atoms and molecules. Statistical mechanics relates the macroscopic property of matter to the underlying microscopic processes.
* Thermodynamics is based on three main laws:
  + Zeroth Law: The zeroth law allows us to define the concept of temperature.
  + First Law: The first law is essentially the law of conservation of energy.
  + Second Law: The second law states essentially that only certain thermodynamic processes can occur.

# The Zeroth Law

## Thermal contact

* Two objects are said to be in thermal contact if heating one of them(ie, adding heat) causes macroscopic changes to the other.
* It is important to note that the two objects do not necessarily have to be in *physical* contact to be in thermal contact with each other. For example, if two bodies of water are separated by a metal plate, the two bodies of water are still in thermal contact as adding heat to one body will cause changes in the macroscopic properties of the other body(e.g, vaporising).

## Thermal isolation

* Two systems are said to be in thermal isolation if heating one system does not cause a macroscopic change in the other.

## Thermal Equilibrium

* Two systems are said to be in thermal equilibrium if they are in thermal contact and there are no macroscopic changes in either body over time.
* Two systems will be in thermal equilibrium if they have the same temperature.

## The Zeroth Law

* The zeroth law states that if a system A is in thermal equilibrium with system B, and B is in thermal equilibrium with system C, then A will be in thermal equilibrium with C.
* The zeroth law is fundamental to thermodynamics because without it, we would not be able to define temperature.
* This is because if the zeroth law did not hold, we would not be able to measure temperature. To illustrate, consider Professor Grasso’s thought experiment whereby a thermometer is placed in thermal contact with system A until thermal equilibrium is reached. Assuming that A is in thermal equilibrium with system B, if we then place the thermometer in thermal contact with B, the thermometer would in fact exhibit a macroscopic change, indicating it is not in thermal equilibrium with B. This then means we can’t say that A and B have the same temperature, even though they are in thermal equilibrium.

# Temperature

* The temperature of a system is defined as the mean translational kinetic energy of the particles of the system.
* Temperature can also be defined as the property common to two objects in thermal equilibrium.

## Instruments for measuring temperature

There are two main instruments for measuring temperature.

* **Thermometers:** Thermometersmeasure the *value of temperature* of a system. Thermoscopes are constructed out of anything whose macroscopic property changes with a change in temperature. E.g, a glass of water.
* **Thermoscope:** A thermoscope measures the *change in temperature* of a system.

## Temperature Scales

In addition, temperature can also be quantised according to the following systems:

* The Kelvin scale: The Kelvin is the SI unit of temperature. Each Kelvin is equal to one °C. **0 Kelvin represents absolute zero, the lowest temperature possible.** The equation for Kelvin in terms of °C is given by:
* The Centigrade scale: The centigrade scale is such that each unit(centigrade) is equal a Kelvin, but the temperature of a system in Centigrade is always 273.15 less than it would be in Kelvin:
* Absolute Zero: At the temperature of absolute zero, the temperature of a system is given by:
* IMPORTANT: Remember that 1°C as a unit is always equal to 1K.

# Thermal Expansion

The phenomenon whereby almost all substances expand or grow in size as temperature increases.

* On a microscopic level, thermal expansion can be described as the increased average movement and separation of the constituent molecules as kinetic energy increases.

## Linear Expansion

* Linear expansion occurs when the length of an object changes as temperature changes.
* The equation of linear expansion is given by:

is the change in length , is the initial length, is the **co-efficient of linear expansion** for an object and is the change in temperature in either °C or K.

* Note that the coefficient of linear expansion depends on the material of the solid body. It has units of K-1.  is usually a very small number in the order of magnitude 10-6.
* IMPORTANT: The length can be any chosen linear length on the body.

## Volume Expansion

Volume expansion occurs when there is a change in volume associated with the change in temperature.

* The change in volume can be expressed by the following equation:  
    
  or

Also note that:

* Here, is the change in volume(m3), Vi is the initial volume, is the **coefficient of volume expansion** and is the change in temperature of the system.
* The coefficient of volume expansion: Again, the coefficient is dependent on the material the solid body is comprised of.

## Thermal Expansion – Thermometers

* As previously explained, all thermometers are created by measuring some macroscopic change. This change is often just thermal expansion.
* **Alcohol thermometers**: Typical alcohol/mercury thermometers rely on the mercury or alcohol undergoing thermal expansion, but to a greater extent than the glass.
* **Bimetal strips**: a bimetal strip is comprised of two strips of different metals fixed together(welded). Suppose we have two metals, metala and metalb, where .
  + As the bimetal strip heats up, each strip undergoes volume expansion/linear expansion, but due to different co-efficients of linear expansion, metala will expand to a greater extent than the other. Thus, the bimetal strip bends away from metala.
  + As the metal is cooled down, metala will shrink by a greater magnitude, so the bimetal strip bends towards metala.

# Gases

## The macroscopic state

* State variables: State variables are a set of macroscopic quantities which together completely describe the thermodynamic state of a substance. State variables are **path independent** as the method by which you used to make each variable equal is irrelevant.
* The state variables of gases in thermal equilibrium are temperature, pressure and volume. Together, these three variables fully describe the macroscopic state of the gas.

## Ideal Gases

* The gas particles obey Newton’s laws of motion.
* The volume of the gas particles themselves is negligible in comparison to the space between them.
* Gas particles travel in random, straight lines.
* The collision between particles is entirely elastic – ie, no energy is kinetic energy is lost in collisions.
* There are no attractive or repulsive forces between gas particles.
* **Note**: A gas can be classified as ideal if the ideal gas law equations hold.
* **Note:** In reality, the ideal gas laws only apply to theoretical “idealised” gases. Real gases behave differently. You often need more complicated equations for relating the state variables.

## Equations of an Ideal Gas

* The first equation is given by:

Where:

* p is pressure(Pa, Pascals)
* V is volume(m3)
* n is the number of moles of gas
* R is the **universal gas constant =** 8.314 (JK-1mol-1)
* T is the temperature (K).
* The second equation is given by:

Where:

* N is the number of gas particles.
* k is **Boltzmann’s constant**, where k = JK-1.
* REITERATION: Any gas for which the ideal gas law holds is an ideal gas.
* Derivation of Avogadro’s number: Combining the two equations for an ideal gas, we get
* **Note:** The ideal gas law is often called the “ideal gas equation of state” as it relates the state variables of p, V and T. It shows that you do not always need to know all state variables to fully specify the thermodynamic state of a substance.

## Derived Laws of Gas

The following laws apply when there is a change in the state variables for an ideal gas.

1. **Boyle’s Law**For a gas of constant temperature and number of moles/particles
2. **Charles’ Law**For a gas of constant pressure and number of moles/particles
3. **Gay-Lussac’s law**For gases of equal volume and number of moles/particles

## Thermodynamic process

The procedure by which you change a system from one state to another.

## Quasi-static processes

A process carried out so slowly such that thermal equilibrium is established at *all* points between the initial and final state.

* For a quasi-static process, there are theoretically an infinite number of different paths from the initial to final state.

## Non quasi-static processes

* In processes which are carried out quickly, thermal equilibrium will not be established and the state variables will therefore not be defined at points between the initial and final state.
* In a non quasi-static process, we cannot measure the state parameters at points in between the final and initial states.

## Isotherms

A process is described as **isothermal** if the process is quasi-static and carried out at a constant temperature.

* An isotherm is the smooth curve consisting of all points on a pV diagram for an isothermal process.

## Reversibility

* A thermodynamic process is reversible if, firstly, there can indeed be a defined path for the process, and secondly, if the path taken is equivalent to the thermodynamic path of the original process but in a reverse direction.
* Equal end and initial states alone does not imply reversibility. It is about the path taken rather than the end points of the process which determine reversibility.
* Thus, reversibility usually entails a quasi-static process.
* If the process is not quasi-static*, then p and V cannot even be defined for points in between the initial and final states, so it makes no sense to think of a path in the pV diagram*.

# Heat

## The system and environment

* The part of the universe we are interested in is named the system.
* The rest of the universe is called the environment.
* The imaginary, two dimensional surface between the system and environment is called the boundary.

## System types

* Open: A system which exchanges both matter and energy with the environment. E.g the human body.
* Closed: A system which exchanges energy but not matter with the environment. E.g a cup of coffee.
* Isolated: A system which exchanges neither energy nor matter with the environment. E.g, perfectly insulated coffee. Technically, this type of system is not found anywhere in nature, and can only be approximated.

## Heat

* Heat refers to the energy being transferred across the boundary between a system and environment due to a difference in temperature.
* The amount of heat transferred *into* a system from the environment is denoted by Q(J).

* The transfer of heat means that the internal energy or thermal energy of a system increases.

Heat isn’t the only kind of energy which can be transferred, work can too.

## Temperature change and Heating

* General formula: The relationship between the heat transferred into a system and the system’s change in temperature can be expressed generally by the equation:  
    
  (JK-1) is a constant – called the **heat** capacity - that depends on the sample.   
  **Note:** Because C has a positive sign, this means that Q and T must have the same sign, ie, if > 0, > 0 and vice versa.
* Standard formula: The relationship between the heat transferred into a system and the system’s change in temperature can be more accurately expressed by accounting for the mass of the system:  
    
  *c* (Jkg-1 K-1)represents the specific heat capacity of the material from which the system is comprised.
* Molar specific heat: The equation linking the no.mols and energy transferred can be expressed as:  
    
  -1 K-1)represents the molar specific heat of the material.
* **Note:** The specific heat capacity of a material can depend on the temperature and also whether it’s the systems volume or pressure which is being changed under heating(there are different specific heats for each scenario). This point is mostly relevant for gases rather than solids or liquids.
* **Note:** If you think about it, Q is really just the change in energy of the system(this will be discussed later through the equation ), so what the equations above describe are in fact the rate of change in energy vs. rate of change in temperature.

## Phase Transformation

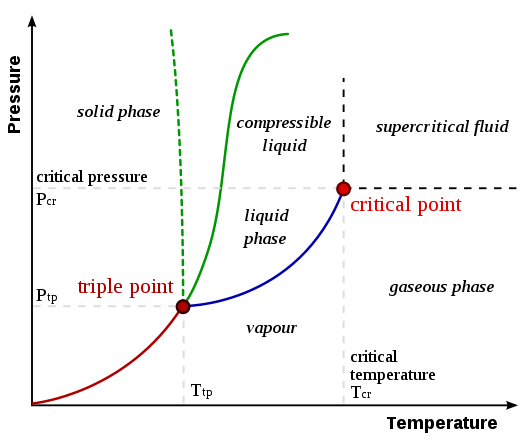
A phase transformation/change/transition occurs when a substance alters its state(e.g solid, liquid, gas).

* Up to now, we have only been examining the effect of heating on the temperature of a system. However, the heat transferred doesn’t necessarily have to increase the temperature of the system – it can also change the phase of the substance.
* The energy transferred can be used to either:

1. Increase the kinetic energy of the particles of a substance.
2. Overcome the attractive forces between the molecules by increasing the potential energy of the molecules.
3. Both 1 and 2(only for impure substances).

* To reach the phase transformation stage, the substance must first reach the phase transition temperature.
* After reaching the transition temperature, all added heat will be used to increase the potential energy between molecules, so kinetic energy(and therefore the temperature of the particles) remains the constant during phase change.
* **Energy added vs phase transition:**system, while m is the mass of the substance and L is the constant called **heat of transformation**.
  + **Note:** The heat of transformation for a substance, firstly, depends on the material comprising the substance, along with the phase transition occurring. There are three phase transitions:
    - Melting/freezing: From solid to liquid and vice versa. The latent heat of transformation is called the **heat of fusion** in this situation and is denoted as Lf.
    - Vaporisation/condensation: From liquid to gas and vice versa. The heat of transformation is called the **heat of vaporisation** and is denoted as Lv.
    - Sublimation/deposition: Solid to gas and vice versa. The heat of transformation in this situation is called the **heat of sublimation** and is denoted as Ls.
  + It is important to note that because both L and m are scalar quantities, the signage in front of the expression mL must be decided upon manually due to the fact that the energy transferred could be leaving or entering the system depending on the scenario.

## Phase Diagram

A diagram showing the conditions under which a substance will be in certain states.

* Notable features:
  + Triple point: A point on the plot under which all three phases are in thermal equilibrium.
  + Critical point: A point at which the boundary between gas and liquid terminates.
* The implications of the phase diagrams are that if, for example, we keep pressure constant, then increase temperature, a substance could move from a solid, to liquid, then gas.
* Likewise, if we keep temperature constant, and steadily increase pressure, this could cause substance to change from a gas to liquid to solid.

## Heat Transfer mechanisms

* Thermal Conduction.
* Convection.
* Radiation.

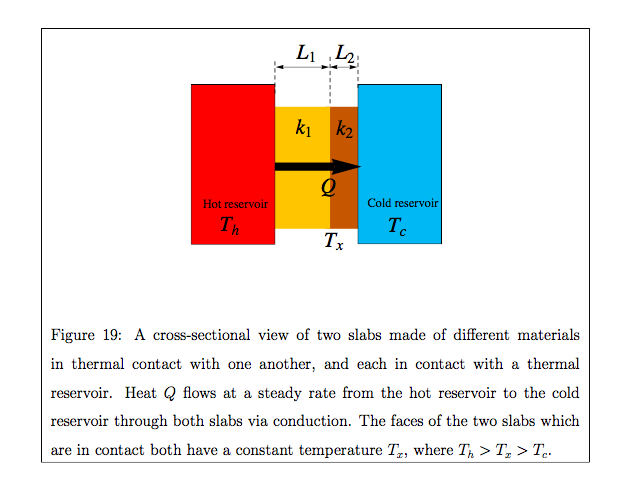
## Thermal Reservoir

Also called a heat bath, a thermal reservoir is any individual body or environment which always maintains its temperature when placed in thermal contact with a system.

* In reality, there is no such thing as a proper thermal reservoir technically, but objects which approximate it have a large specific heat capacity.
* The idea of a thermal reservoir is important for calculations of conduction rates and emission rates, because it means that the temperatures of the body remain constant, so solving the equations are easier.

## Conduction

* The transfer of energy through physical contact.
* On a microscopic scale, conduction occurs because particles of a higher kinetic energy(from the body with greater temperature) will transfer energy to particles of lower energy(from the body with lower temperature) through collisions.
* **Equation:** The general mathematical equation relating the rate of conduction between two bodies is given by  
    
  k (JK-1 m-1s-1) is a coefficient of thermal conductivity, varying for the material of the medium of conduction. A is the surface area of the conducting face, L is the length between the two objects, Th is the temperature of the hotter object and Tc is the temperature of the colder object.   
    
  We can also define the thermal resistance of the conductive medium as
* Conduction over multiple mediums: Suppose we have two slabs in between our thermal reservoirs as shown.

Now, assuming that the system is in a steady state – ie, temperatures for any fixed point within one of the slabs will be constant over time, then the rate of heat transfer must be constant through both slabs, since otherwise, energy will accumulate in a single slab, causing the system to move out of a steady state as temperatures change.

Thus:

We can therefore use substitution to find H as:  
  
  
 more general sense(for *n* slabs):

## Convection

* Fluid: Fluids are defined as liquids or gases.
* Convection can only occur in fluids.
* Procedure:

1. Some part of the fluid comes into contact with a hotter body.
2. The fluid is therefore heated via conduction and its density therefore decreases.
3. The fluid therefore rises above other parts of the fluid with greater density, so colder parts of the fluid then come into contact with the heat source and the cycle repeats. This cycle is called the convection current.

* **Note:** convection can only occur when buoyant forces are present. In the present of weak gravitational fields or if the fluid is in free fall, convection cannot occur due to a lack of buoyant forces.
* Cold convection: Convection with a cold current can also occur if the cold reservoir is at the top of a warmer fluid.

## Radiation

Thermal radiation is the radiation exchanged between two bodies – or a system and its environment – due to a temperature difference.

* Because radiation is in fact electromagnetic waves, radiation does not need a medium to propagate.
* **Emittance:** The equation describing the rate of heat emission by an object(a) is given in the Stefan Boltzmann law  
    
  Pem (W) is the power of emission, ie, rate of radiation transfer, e is the emissivity of the object from 0 to 1(unitlesss), is the Stefan-Boltzmann constant, A the area(m2) of the emitting object and T the temperature(K) of the emitting object.
* **Absorption:** All objects will also absorb radiation (except perfect reflectors). The rate of energy absorption by the same object(a) is given by   
    
  Where Pabs is the rate of energy absorption via radiation, and T­env is the temperature of the environment. **Remember:** A is still the surface area of the same object which was emitting. In other words, e, are the same for both the emission and absorption equations.
* The net radiation absorption rate is therefore given by Pnet = 0, then no energy will be lost or gained, so the object and environment must be in thermal equilibrium.
* Blackbody: A black body is any object with e = 1, and will thus be a perfect absorber and perfect radiator.
* **Note:** The rate of radiation is often given in units of W/m2, this indicates that if you want time for example, you need to divide the amount of energy by the power multiplied by the area:

Note: Carry on answers use the original, unrounded answer from previous questions.

# The First Law of Thermodynamics

The change of internal energy in a system is equal to the sum of heat added and work done on the system, and depends only on the initial and final states of the system and not of the process involved.

## Work

The mechanical transfer of energy due to a force acting over a displacement.

* The work *done on a system* is denoted by the symbol W.

## Path Independence

When the path by which a quantity or variable takes a certain value is irrelevant.

## Thermodynamic process – First Law

* As previously discussed, moving a system from an initial to final state can involve energy transfer via two means, heat and work.   
    
  transferred *into* the system while W is work done *on* the system.
* Moving a system from an initial state to final state can be brought about through an infinite number of thermodynamic paths, with an infinite number of combinations of Q + W.
* However, in all cases, the quantity Q + W will always be observed to be the same, because   
    
  f and Ui are path independent(in ideal gases, only dependent upon the temperature), Q + W will only depend upon the final and initial states rather than the path taken. To illustrate, consider this pV diagram illustrating the thermodynamic path taken by a gas
* **Note:** Though the quantity Q + W is path independent, **quantity Q and quantity W by themselves will be path *dependent***, since each different path will have different values of Q and W (even if their sum is still the same). In other words, even if a system moves from the same initial to final state, **the sum of work done and heat transferred will not be the same for different paths.**
* The fact that the sum of Q + W will always remain constant for the same initial and final conditions indicate that they measure the change of an intrinsic or internal property of the system – a system called the internal energy of the system.

## The internal energy of a system (U)

The internal energy of a system, denoted by the symbol U, refers to the sm of kinetic and potential energy stored within a system.

* Having defined the property of a monatomic gas as one which contains no potential energy and only kinetic energy, we can therefore determine the total internal energy of the gas through an earlier proof for the total kinetic energy of the gas:
* In other words, we say that the internal energy of a system is a function only of its temperature and is independent of all other variables.

## The First Law of Thermodynamics

The first law of thermodynamics states that the change in internal energy of a system is the sum of heat added to and work done on the system, and depends only on the initial and final states of the system and not on the particular process involved.

* The first law can be summarised as follows:

Note that this equation in fact holds for all thermal processes, meaning that the internal energy of a monatomic gas will only change if its temperature changes.

* Zero change: From the equation above then, we can see that if there is no change in temperature, then there will be no change in internal energy, even if pressure and volume changes. The reason as to how this can happen given that volume changes cause work to be done, is due to the fact that heat energy must be transferred in/out of the system to compensate. In other words  
    
  As we will see later, this equation applies for a isothermal process.
* Infinitesimal change: Given the above equations, we can therefore write the first law in infinitesimal form

## Work done on a Gas

For a gas moving from state I to f through some any quasi-static process, the work done on *any gas* as it undergoes *any* kind of quasi-static volume change is given by

* From the above equation then, we can also assert that **the work done on a gas is equal to the area under the curve of a pV diagram from Vi to Vf**. This method is useful when we are not given the scale for a pV diagram and the change in pressure and volume is expressed as a scalar of some quantity.
* The negative sign indicates that for an increase in volume, work done on a gas is negative (i.e., the gas does work on its environment).
* Cycle: In special cases of a cycle, where the system returns to its initial state, then the work done on the system is given by the area enclosed by the thermodynamic path of taken.

## Universal equation

For any thermodynamic process involving any ideal gas, the change in internal energy is given by

* This idea is given because since the change in internal energy is independent of the path taken, we can calculate the change through a path whose energy input/output we can easily define.
* This is done by imagining a thermodynamic process whereby the first step occurs at a constant volume while the temperature changes. Since work equals 0 (as we will discuss later), the change in internal energy is given by
* The second leg is then isothermal, the path going down that of an isotherm. The change in internal energy is therefore given by   
    
    
  and because there is no change in temperature
* Thus, overall, the change in internal energy is given by

## Constant volume (Isovolumetric or Isochoric) processes

* The change in internal energy for a system undergoing an isochoric process is given by
* This is because for an isochoric process, the work done on a system will always be equal to 0, because dV is 0. In other words, because there is no change in volume, the displacement of caused by the force of the gas particles is 0, so work is 0. Instead, the change in internal energy is caused by heat transfer to/from the system. We define the heat transferred as
* Equating this with our equation for the total energy of a monatomic ideal gas then, we say that  
    
  Note that we will derive the equations for CV of non-monatomic gases later on.

## Constant Pressure (isobaric) processes

* The work done on a system undergoing an isobaric process is given by
* Derivation: Since V can be expressed as a function of p, we can therefore derive the following equation
* Other equations: However, while work is done on the system in this process, the change in internal energy is still defined by the same variables, and the heat transferred is given by the equation  
    
  we use Cp instead of CV.
* Therefore, we can express the three equations as
* The above equation in fact holds for all ideal gases.

## Constant Temperature (isothermal) Processes

* Since , if the temperature is held constant, then there will be no change in internal energy, so Q + W = 0.
* As such, we say that
* We can thus determine Q through finding W, which in this case is given by
* Therefore

## Adiabatic Process

A process where no heat is transferred between a system and its environment.

* The law for ideal gases, pV = nRT, still holds for adiabatic processes, but in addition, we can use the formula
* The above formula can also be expressed in the form
* Deriving for temperature:  
    
  And thus, if the final and initial temperatures are known
* Deriving for work:  
    
  Using , we can insert this into the work equation of p to get

## Degrees of Freedom (f)

The minimum number of parameters of co-ordinates needed to fully specify an object’s location and orientation in space.

* Translational degrees of freedom: degrees of freedom associated with the x, y and z co-ordinates.
* Rotational degrees of freedom: degrees of freedom or parameters which correspond to the rotation of an object.
* Point: Objects which are points or spheres, e.g monatomic molecules, only need 3 parameters to define their position: x, y and z co-ordinates. Note that monatomic molecules do not require orientation parameters, since revolving one would not create any geometrical difference.
* Diatomic: Diatomic molecules, shaped like dumbells, require 5 parameters to define both their position and orientation. This includes their x, y and z co ordinates, as well as their x and y rotational orientations. The latter two are known as **rotational degrees of freedom**.
* Polyatomic: Polyatomic molecules require six parameters to define their position and orientation.
* **Note:** We can also include modes of vibration, and this will increase the number of degrees of freedom, but will not be counted in this unit.

## Equipartition of Energy

A theory which states that for a given amount of energy gained by a particle, the energy will be divided equally between the particle’s degrees of freedom.

* In other words, the theory states that the added energy will be divided equally amongst the different ways internal energy can be stored by the gas.
* On average, a molecule of gas will have an energy given by:
* Therefore, the total internal energy of a gas, U, is given by
* Therefore, the equation for the change in internal energy of a gas is given by:

## Free Expansion

A process that is both adiabatic and where no work is done.

* This implies that since Q = 0 (adiabatic) and W = 0, the **change in internal energy is also zero.**
* Since the change in internal energy is zero, this means that the **temperature of the system remains constant**.
* Interpretation:
  + Kinetic energy: The first way of thinking is that particles of an ideal gas move apart, but do not change the internal energy they carry.
  + Temperature: The second way of interpreting free expansion is by thinking of a scenario in which the particles of an ideal may move apart, but their kinetic energy remains constant. Since the gas is ideal, temperature is only proportional to kinetic energy, which therefore remains constant.
* Example: An example of a free expansion scenario might be one in which a balloon pops in a vacuum. The gas inside won’t do work on anything, and no heat is transferred since the process is too fast anyway (and also, there is nothing to exchange heat with). Another example might be the opening of a valve in an isolated chamber, thus increasing volume.

## Pressure-Volume Graphs

* Work: For any thermodynamic process, the amount of work done ***by*** the gas is given by the area under the curve (for any process) assuming the volume is increased. If the volume decreases, then the area becomes the work done ***on*** the gas.
  + Cycle: For a cycle, ie, a process whereby the gas ends in it’s initial state, the work done on the gas is given by the area encircled by the path.
* The change in internal energy is given by:
* **Remember:** Always check the notation of your answers: if W < 0, this means work was done ***by*** the gas. If W > 0, work was done ***on*** the gas. Your formula for working out W is work done *on* the gas while the area beneath the pV curve is work done *by*the gas.
* For any (increase/decrease) in pressure/volume while volume/pressure is constant, temperature will increase/decrease.

# Second Law of Thermodynamics

## Reversibility

* Spontaneity: A process is said to be spontaneous if it does not require any significant effort to occur. In terms of states, we say that a system will move to a certain state without the need for a significant amount of energy.
* Examples: To illustrate, we never for example, see a gas move from being diffused to arranging itself in a single corner of the room, unless we exert significant force onto it.
* Therefore, we say that a process is reversible if it can spontaneously take a system back from the final to initial state.

## Entropy (S)

A state variable which measures the magnitude of disorder in a system

* As the disorder of a system increases, ie, the number of ways it can be possible arranged, the entropy will increase.
* Probability: The entropy of a system can also be interpreted as the probability the system will be in its current state. Typically, because there are only few organised states, a system is far more likely to be in a disorganised state.
  + Disorder: Disorder represents the number of ways in which we can arrange the particles of a system but still have the system represent the same thermodynamic state.
* Entropy also measures the amount of useful work a system has to offer.

## Entropy Change (J/K)

The entropy change for a system is given by  
  
f is the final entropy of the system, Si is the initial entropy of the system, dQ the incremental change in heat transferred, and T the temperature of the system over which the entropy change occurs. If temperature is not constant, refer to the equations given below. **Remember:** Entropy change will only be non-zero iff the process is irreversible.

* In infinitesimal form, the above equation is expressed as
* If the temperature of a system is not constant, then we have to substitute the infinitesimal form of heat transfer into the equation, giving
* Word form: In an isolated system, if an irreversible process occurs, the entropy of the system must increase.
* **Note:** It should be remembered that within a system, a particular subsystem can have an entropy decrease, but the corresponding entropy increase in other subsystems must be greater than the decrease to maintain the second law of thermodynamics.
* Generally speaking, an increase in temperature of a system means an increase in entropy.

## The second law of thermodynamics

In an isolated system, the entropy cannot decrease. If a process occurs in an isolated system, the **entropy change of the system will be zero if the process is reversible**, and will be greater than zero if irreversible.

* The second law can be expressed as, for an isolated system undergoing some process, the entropy change of the system is given by

Mechanics

# Introduction

## Standard Units (SI Units)

* Length: Measured in meters, denoted by “m”.
* Time: Measured in seconds, denoted by “s”.
* Mass: Measured in kilograms, denoted by “kg”.

## Particle Motion

* Position:
* Displacement:
* Velocity
* Acceleration:

## Finding Acceleration (from illustration)

* To find the **direction** of acceleration at a point, we have to think about the direction and magnitude of its velocity at point immediately before that point. We then consider the direction and magnitude of its velocity at the point, and consider what direction this velocity has changed.

## Special Cases

* Constant velocity:  
    
  If   
    
  Then   
    
    
  Where r is the displacement in the direction of velocity.
* Constant acceleration:   
    
  , by integrating, we find  
    
  Note however, that in this case, the vector r is not the same as position, but rather, the displacement. Thus, to find position, we use

Essentially, to find position, all we do is add the displacement to the original displacement.

## Newton’s Laws of Motion

1. An object will continue in a state of rest or constant motion unless acted upon by a net non-zero external force.
2. The acceleration of the object is the direction of and proportional to the net force. This law can be expressed mathematically as
3. For every action, there is an equal and opposite reaction.

## Parabolic motion

* Equations
* Range: Maximum range is achieved when the angle of initial velocity is 45 degrees above the horizontal. As the angle of initial velocity approaches 45, the range of a projectile approaches the maximum.
* Maximum Height: Maximum height occurs is only dependent on the initial vertical velocity – the greater the initial vertical velocity, the greater the maximum height.
* Time of flight: Again, only dependent on initial vertical velocity due to the equation
* Time to maximum height: It should be noted that the time for a projectile to travel to its maximum height, assuming no air resistance, is the same as the time for the projectile to travel back down.

## Forces

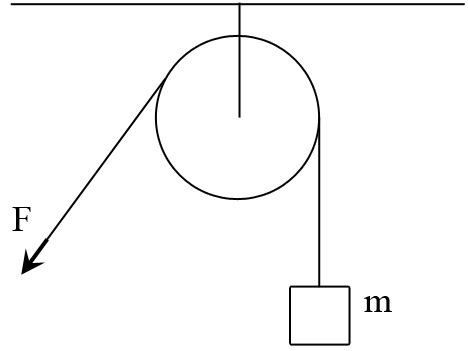
* Normal Force (N): The normal force is a contact force between two surfaces. In the majority of cases, the normal force should counteract the gravitational force acting on an object such that:
* Frictional force (Ff): There are in fact two types of frictional forces. The first is static friction, and the second is dynamic friction. Static friction is defined as a force opposing the onset of motion for an initially stationary situation. Static friction can be found by:  
    
  µs is the coefficient of static friction, and N is the normal force. However, we must **remember** that static friction in fact changes: as the force applied increases, so too will static friction. The maximum value of static friction however, is given by the equality, ie the product of the static coefficient of friction and Normal force.   
    
  Dynamic/Kinetic friction can likewise be defined as the force opposing existing motion. Kinetic friction can be found by the equation:  
    
  This means we often experience a jerk as the object starts to accelerate. However, **remember** that while the static friction changes, kinetic friction remains at a constant value defined by the above equation.
* Tension (T): The intermolecular restoring force which occurs when atoms/molecules are pulled too far apart. There are a few key aspects of tension forces. The first is that tension is essentially transferred for stationary objects, but only due to Newton’s third law. In the case of a moving system of objects, the force applied on each object in the direction of acceleration will vary.
  + A special case: In the majority of questions, the string or object which exerts tension will be massless, so we can find the tension for the whole object very easily. However, when the string/object has mass, then we say the tension at point x from the suspension point of the string/object is given by:  
      
     In other words, when the rope itself has mass, the tension at a point is equal to the weight of the remaining rope at the bottom. Therefore, when a mass, M, is hung from that object, tension is given by:

## Conservative forces

A force for which the work done by it or against it is path independent and depends only on the start and end points. It is only the change in potential energy which has any significant meaning.

* Central forces: Any for law of the form F(r) where there is no dependence on theta will be necessarily be conservative and has an associated potential.

## Pulleys

* It is important to remember that there is no set positive or negative directions in a pulley system.
* This is because as long as you are pulling/pushing, then the direction of the applied force does not matter, only the magnitude. In other words, we can change the direction of the force, but the magnitude will remain the same.
* For example, consider the system below:

Here, even though we could change the angle at which we apply the force F, this would not change the magnitude of the applied force because all that matter is that we are either pushing/pulling with the same magnitude of force.

* As such, direction in pulleys shouldn’t be defined in terms of typical x, y or z orientations, but rather, in the direction of pull/push.

## Solving problems in mechanics

1. Define your directions in a way that suits the context.
2. Check all values are given in SI units. If not, convert them to SI units.
3. Think about which equation holds the unknown variable.
4. Express the unknown in terms of other variables.
5. Solve for the unknown.

## Overcoming friction

* With regards to questions involving motion and friction, it is important to remember for the net force on an object, an inequality occurs when we have static friction, whilst an equality is needed for continued motion. This is because for a stationary object to undergo motion, according to Newton’s first law, there must be a net non-zero force applied to the object for it to accelerate, so the applied force must be greater than the static frictional force. On the other hand, when we have an object moving, to keep it moving, all we need to do, according to Newton’s first law, is to have a zero or positive net force acting on the object in the direction of its motion, so the applied force only needs to equal the frictional force.

# Rotational Motion

## Conventions

* The positive direction is typically the counter clockwise direction.

## Angular motion

* Position:
* Rotational Velocity (radians/second or degrees/second):   
    
   is the angular velocity, is the rate of change in angular position, or the rate of change in the angle of rotation. The units of angular velocity are **radians/second or degrees per second**.
* To find the number of **revolutions** per second, we must divide the angular velocity by radians or 360 degrees.
* Note that the rotational velocity in fact is not on the plane of rotation, but perpendicular to the plane of rotation. As we will see later, we can also derive the equation for tangential velocity (the velocity of the centre of mass) from the equation for rotational velocity.
* Rotational Acceleration:   
    
  linear acceleration tends to be in the direction of linear velocity, unless the direction of linear velocity is changing.
* Acceleration (radial):

**Note:** From PHYS1002, if we’re asked for the equation a = , that implies radial acceleration. If we’re asked for , this implies that we’re talking about tangential acceleration.

## Linear Derivations from rotational motion

* The following equations for linear motion can be found through the equation for angular motion.
* Tangential velocity:
* Tangential acceleration:

## Non-Slip scenarios

* **Non-Slip scenarios**: In many instances, we will often be asked to determine a value if a ball is rolling without slipping. In short, this simply means that the ball’s linear velocity, ie, com velocity, is equal to its tangential velocity.
* This is because, by definition, if a ball is not slipping or skidding, then its infinitesimal point of contact with the surface on which the ball is moving must not be moving relative to the floor. In other words, while the centre of mass of the ball or circular object may be moving relative to the floor, its point of contact will not. Thus, recalling the formula for relative velocities, we can express the relationship between velocities as thus:

And because   
  
  
**the magnitude of the centre of mass velocity will be equal to tangential velocity, but the com velocity will be in the opposite direction to tangential velocity.**

* **Remember: Non-slip means that the com velocity is equal in magnitude but opposite in direction to tangential velocity.**

## Rotational Momentum

* As we have seen, essentially all linear concepts of motion such as position, velocity, acceleration have analogues (equivalents) in a rotational sense. This is the same for the idea of linear momentum, whose analog is rotational momentum.
* Rotational momentum is given as:  
    
   momentum of a particle at a position r with respect to some point. p is the linear momentum of that particle. Note that the angular momentum is the vector cross product of the angular position of a particle and its linear momentum.   
    
  And since
* In our idea of linear momentum, momentum was expressed as the product of mass (the inertial feature) and linear velocity. The rotational analog of the inertial property is given in the next topic.

Rotational Inertia  
In general, rotational inertia can be expressed as:

where I is known as the rotational inertia of an object, m is the mass, and R is the **distance** of the point mass from the rotation axis. This means that if a mass is already on the rotational axis, then its rotational inertia is 0.

* **Remember:** The axis of rotation is 1 dimensional but is not a point – it is a line. The distance R represents the shortest distance from the point mass to the axis of rotation.
* However, we must calculate rotational inertia based on each context. In the case where we are calculating the rotational inertia of a discrete number of point masses around an axis of rotation, we use the formula:
* In the case that we have an object consisting of continuous numbers of point masses, to calculate rotational inertia, we take the integral:  
    
  we often have to find an expression for dm in terms of . This then allows us to perform the integration to determine the total rotational inertia of an object.
* Parallel Axes: Given a certain value for the moment of inertia around one axis of rotation, we can then find the moment of inertia around another axis of rotation which is parallel to the original axis. The inertia can be found through the equation:  
    
   is the moment of inertia about the new rotational axis, the rotational inertia about the original axis, m the total mass of the object, and is the distance of the new rotational axis away from the original axis.

## Torque

The turning effect due to a force applied about a rotational axis. Torque is defined by:

Where , torque, is the vector cross product of a force F acting a distance r from the pivot point. Note that the direction of r is away from the pivot point towards the point where the force was applied. Torque can also be defined as the rate of change in angular momentum: thus, the direction of torque also describes the direction of the change in angular momentum.

* Derivation: We can also derive the following equation from the above definition of torque:

Where is the rotational acceleration of a mass.

* Torque can be thought of as the rotational analogue of force, where I is the inertial property and the rotational analog of linear acceleration.
* Conservation of rotational momentum: As noted, because torque represents the rate of change in rotational momentum, if there is no net external torque applied to a system, then the system’s rotational momentum will be conserved. Thus, if we increase the mass of a rotating system or increase the distance each point mass is from the axis of rotation, then for rotational momentum to be conserved, the rotational velocity must decrease and vice versa.
* Implication: Thus, for a given force, the turning effect of that force will be proportional to the distance that force is applied from the axis of rotation. Thus, as the distance the force is applied to an object from its rotational axis increases, so too will the turning effect.

## Pulley Scenario with only one Mass

Reference: Mechanics 2, Assessed, Question 4.

* The idea behind this question is that . We therefore find the angular acceleration by rearranging the equation to get . Here, r simply represents the radius of the pulley, F is the tension force the string exerts on the pulley, and I is simply the rotational inertia of the pulley. Because , we cancel the r on the top to get  
    
   (this can be proven independently)

Hence

* **Key Points:** The torque generating force is the one which is applied to the rotating object at a certain point. The key step is equating rF to . We then convert F to T, and T in turn to an expression of acceleration, and acceleration to .

## Rotational-Linear summary

|  |  |  |
| --- | --- | --- |
| Property | Linear | Rotational |
| Position |  |  |
| Velocity |  |  |
| Acceleration |  |  |
| Momentum |  |  |
| Inertial property |  |  |
| Force |  |  |
| Energy |  |  |

# Energy, forces and momentum

## Total Kinetic Energy

* Whilst before, we have only kinetic energy in terms of linear kinetic energy, we must now account for rotational kinetic energy through the formula:
* In terms of the conservation of energy, we still use the typical formula, but now using total kinetic energy instead of just linear kinetic energy:  
    
  , as in the case of the test. Currently, it seems that a net decrease in kinetic energy results in a decrease of rotational kinetic energy and vice versa. In fact, because omega can be described by v, even if we were to “exclude” rotational energy from the conservation of energy equation, because v itself would decrease, and v is proportional to omega, then omega would also decrease.

## Energy of a Pendulum

1. Assuming that the pendulum is stationary when it is height h above its minimum vertical displacement, in its initial state, the pendulum would have total internal energy of .
2. However, once the pendulum swings to its lowest vertical displacement, we use the conservation of energy law:
3. So, because the pendulum would have no potential energy at the point of lowest vertical displacement (in comparison to its reference initial point), we have:
4. Therefore, we rearrange to find velocity:

# Air Resistance (N)

Here, the coefficients depend on size, shape, surface roughness and fluid properties of an object.

* In general, the coefficients can be found for a smooth sphere in air by:

## A special case

* Referring to Mechanics, Lecture 8, example 2, for figuring out the drag force on an object after it has fallen a certain distance, we act as if the drag force had not been acting on the object for the distance it had travelled before the point where we try to determine the drag force acting on it.
* Otherwise, we would be unable to calculate the velocity of the object at the point where we try to determine the drag force.

Waves and Optics

# Simple Harmonic motion

## Displacement

Here, the phase is given by , while the **phase angle** is given by .

## Velocity

## Acceleration

## Simple Pendulum

L represents the length of the string, while g represents the acceleration due to gravity on earth, and T represents the period of oscillation for the pendulum.

# Wave Motion

wave (maximum displacement from the equilibrium position). If the operation is a +, then the wave must be travelling in the –ve x direction, and if the operation is -, then the wave is travelling in the +ve x direction. Omega represents the angular frequency, which is while k is the wave number.

**Note:** The above formula only works under the **small angle approximation**, ie, we assume period is independent of amplitude only under the small angle approximation. If the angle is large, the period is proportional to the amplitude, and so the frequency is inversely proportional to amplitude.

## Wave Speed

* Generic:
* On a rope:

Where F is usually T, the tension in a rope.

* Given period and wavelength (same as generic essentially)

Phase Difference   
To calculate the phase difference between two waves, simply look at the x values (generic x, in terms of axes names) of two points on the waves which are equal in terms of y. Find the difference between the x values, and determine the difference as a proportion of wavelength of period.

Standing Waves  
Standing waves occur when two waves **of the same amplitude, frequency and wavelength but travelling in the opposite direction superimpose**. This results in the wave merely **oscillating in a fixed position without travelling.**

* Both ends open: The potential wavelengths for a medium of length is given by:
* One end closed, one end open:
* Both ends closed:
* Mathematical description:

# Diffraction and Interference

## Multiple Slit interference

represents the angle between plane of the slit and the line which runs perpendicular to from the beginning of to some point on .

* is an integer which represents the order of the maxima, with the centre maxima representing the zeroth order. The maxima of the order are **equidistant** from the centre of the slit axis on either side.
* simply represents the wavelength of the waves passing through the apertures.
* represents the distance between the apertures.
* If the distance between the screen and apertures is large relative to the distance between the maximas, then we may say that  
    
  Here, y represents the distance between maxima, D represents the distance between the apertures and the screen, and represents the angle between the line drawn from the center of the apertures to the maxima point and a line perpendicular to the axis of the apertures.
* However, if the distance between the maxima or fringes is significant relative to the distance from the aperture to the screen, then we have to use:

## Single Slit Diffraction

* We use the same formula as multiple slit diffraction, but now, the equation describes the position of the **minima** rather than **maxima**.

## Doppler Shift

* Wave source approaching observer:

Where is the frequency observed by an observer who is said to be stationary, f is the frequency observed by the source, v is the speed of the wave, and is the speed of the source relative to the observer who is said to be stationary.

* Source moving away from observer:   
    
  Same notation as before.

## Polarisation

Ef and Ei are the final and initial amplitudes of the light respectively, theta is the angle of the polarisation plane, and alpha is the initial angle of the incident light, both angles measured with respect to some origin.

* The formula for the intensity of light is given by  
    
  amplitude of light.
* **Note:** The process of polarisation can be carried out multiple times, with the same generic formula as specified above, and each time, the original angle of polarisation becomes “alpha” while the new angle of polarisation becomes theta.

Reflection and Refraction

* Change in speed of light:
* Change in angle of ray:
* When light is refracted, **only its wavelength and velocity will change, whereas its frequency remains constant.**

# Optics

## Lenses

displacement of the object or image source from the mirror, s’ represents the displacement of the image from the mirror, and f represents the displacement of the focal length from the mirror.

* **Remember:** Each of the displacements have a polarity such that when they are on the same side of the lens as the image source, then they are **negative**. If the variables are on the opposite side of the image source, they are **positive.**
* We then relate the variables s and s’ through the equation:

Electrical Physics

# Fundamentals

* All charge is quantised, with the fundamental charge being that of an electron:  
    
  Here, C denotes the SI unit for charge, Coulombs. **Note:** e is also referred to as the quantum of charge.
* All charged particles will have a magnitude of charge that is an integer quantity of the fundamental charge:

**Direction of Force**

* As we will see later on, the force exerted by one charge on another is dependent on a displacement vector,  .
* **The vector represents the displacement from the affecting charge (q1, the charge which exerts the force) on the subject charge (q0, which experiences the force).**
* Therefore, whether the force is positive or negative depends not only on the polarity of the charges, but also the direction going from q1 to q0.
* The general rules are:
  + **If going from q1 to q0 is in a positive direction**:
    - If
    - .
  + **If going from q1 to q0 is in a negative direction**
    - If
    - If

## Conservational Laws

1. **Charge Conservation**: Electrical Charge is conserved – the total charge in an isolated system is fixed and constant. This is a global conservation law.
2. **Charge continuity**: Charge must flow smoothly from place to place. This is a local conservation law. This implies that the amount of charge flowing into a small region must be equal to the sum of the charges accumulating and leaving that same region.

* **Note:** Charge continuity will imply charge conservation, but charge conservation does not necessarily imply charge continuity.

## Coulomb’s Law

The force acting on a charge, due to another charge, , at a distance r is given by:

Here, represents the permittivity constant, given as:  
  
 represents a unit vector whose direction is from the charge q1 to q0.

* Expressed in magnitude form, Coulomb’s law can be expressed as thus:  
    
  vector form, if both charges have the same polarity, then q0qi would be positive, so the direction of the force would be that of , ie, away from the charge qi (repulsion).

# Superposition

## Principle of Superposition

* The net vector acting on an object or at a point in space and time is given by the vector sum of all vectors acting independently on that object or point.

## Component Method

1. Find an expression for where A represents one of the vectors acting on the point or object.
2. Find an expression for , where A again represents one of the vectors acting on the point or object.
3. Find the sum of the components for all the vectors. Then collect these values into vector form.

## Addition Method

1. Find the expression for each vector’s unit vector.
2. Therefore, re-define the expression for each vector.
3. Then, either add the vectors straight away, or first convert them to vector component form before adding them.
4. **Note:** From past papers however, it seems that positions to the **left of the origin are in fact positive….**

## Force Superposition

**Remember:** From past examples, the position to the left is always positive, while those to the right are negative

**Principle of Superposition (PoS)**:

The net force acting on an object is the vector sum of the forces due to all other charges, independent of each other.

* This means that, for a fixed charge which we are interested in, q0, if we are given two other charges q1 and q2 of equal magnitude and polarity, then the charge (between q1 and q2) which is closest to q0 will exert the greatest force on q0. Thus, the net force will be in the direction of the force exerted on q0 by the closest charge.
* The logical conclusion to this is that if the charges q1 and q2 are the same distance from q0, then there will be no net force.

**Vector form**

From the above equations, we can in fact derive another equation based on the fact that the unit vector is actually the quotient of the vector and its magnitude.  
  
equation in the principle of superposition, we can derive the header equations. We can then find net force by simple vector addition after that.

# Electrical Fields

A vector field defined by the charge and force acting on that charge.

## Electrical Field – Force

If an object with charge q is placed in an electrical field at a point with field strength E, the force experienced by the charge will be of the magnitude given by  
  
fore, the proper equation is given by:  
  
 represents the force vector exerted on the charge, q0. q0 is simply the charge on which the force is exerted.

We can therefore derive the equation:

## Electrical Field Superposition

**Principle of Superposition (PoS):** The net electrical field acting on an object is equal to the vector sum of all electrical fields acting on the charge at a particular point in time and space.

**Vector Form**

The net electrical field at a point p (acting on an object of any charge, since q0 is irrelevant in the equation of electrical fields) is given by:

Again, we can derive this equation due to the definition of the unit vector, and so, we can simply find the above equation by dividing the equation for force by q0.

# Scalar – Vector Fields

Please see appropriate section in [Fundamental Mathematics](#Vector – Scalar) at the beginning of this document.

# Charge Distribution

**Continuous Charge distribution**

Charge is spread out over regions of space in a continuous or smooth manner.

**Uniform Charge distribution**

Where charge is spread evenly over a region.

## Types of Charge distribution

**Uniform Volume Charge distribution**

Total charge(q) divided by total volume(V)

uniform volume charge density.

**Uniform Area Charge distribution**

Total charge(q) divided by total area(A).

Where sigma represents uniform area charge density.

**Uniform Line Charge distribution**

Total Charge(q) divided by total length (L).

Where lambda represents the uniform line charge density.

## Conductors in Equilibrium

Occurs when there is no net motion of charge. This in turn requires 5 conditions:

* The electric field is zero everywhere inside the conductor.
* All charge resides on the surface.
* The electric field is normal to the surface.
* Charges tend to accumulate at sharp points.

# Electrical Flux

## Surfaces

A surface is any imagined two-dimensional membrane which extends through space.

**Closed Surface**

A closed surface has no edges and fully surrounds a **volume** of space.

* Therefore, an imagined surface which has **edges, such as a piece of paper**, would not be closed.

**Gaussian Surface**

A Gaussian surface is any closed surface.

**Orientated Surface**

An orientated surface is a surface where the positive direction has been defined and indicated by an arrow.

* For a closed surface, we always choose the direction from the **inside to outside as positive.**
* This means that if the net flux through a surface is going in rather than out, the net flux is negative.

## Calculating Flux – Counting

leave the specified area, while b represents the number of times the flux lines enter the area.

## Flux for a constant Electrical Field

Where theta is the angle between the area vector, , and a line perpendicular to the electric field. E and A represent the magnitudes of the electrical field and area of the flat surface respectively.

1. Overview: Conceptually, the size of the electrical flux through a flat surface is equal to the magnitude of the electrical field multiplied by the area of the surface.

* **Area Vector :** In previous iterations of the equation for electrical flux, we simply had the formula . The problem however, is such a formula doesn’t take into account the orientation of the electrical field relative to the defined positive direction. As such, we account for this by defining an area vector, , which is perpendicular to the flat surface A, **always** oriented in the positive direction, and whose magnitude is equal to the area of the flat surface A.
* As such, the costheta takes into account the orientation of the electric field: Because the area vector always points in the positive direction, the angle between the electric field vector and the area vector will therefore determine whether or not flux is positive or negative:

## General Flux Equation

* We split a given surface into infinitesimally small chunks, dA. Although the overall surface is uneven, because each area is infinitely small, they are relatively flat.
* Each area therefore has a small area vector, . This vector is perpendicular to the plane of the small area, points in the positive direction and has a magnitude dA which is equal to the area of the small area.
* We then calculate the electrical flux, , for the small area, through the formula:  
    
   represents the value of the electric field at the point in space where the small area is located.
* We then add up the electrical flux for each small area on the surface. Since we’re dealing with infinitesimals, we take the integral over the entire surface, leading to the equation above.

**Gauss’s Law**

Here, the circle on the integral indicates that the integration is occurring over a closed surface.

* **Note**: This equation is only applicable to Gaussian surfaces.

## Gauss’s Law – Electrical Fields

### Derived equation for Electrical Fields (from Gauss’s Law)

**Outside the Sphere**

the sphere essentially acts like a point charge, such that the equation for the electrical field is equivalent to that of a point charge.

**Inside a Uniformly Charged Sphere (Solid)**radius of the sphere, and r is the distance from the centre of the sphere from which we are measuring the electrical field.

**Inside a hollow sphere:**

### Electrical Field Due to a charged line

Electrical Field due to a charged sheet

Electrical Field due to a conducting charged sheet

## Flux - Electrical Fields - Conductors

* **Charge:** There is no net charge inside. Any net charge is on the surface.
* **Electric Field:** The electric field inside a conductor in electrostatic equilibrium is **zero**. The electrical field only exists outside the conductor and on its surface.
* **Electrical Potential:** Every point inside a conductor and on its surface are equipotential.

## Flux and Charge in a Sphere Example

Referring to question 7 of Worked Examples 4.

# Electrical Potential

Electrical potential, V, is a scalar field associated with a given charge distribution.

## Work – Electrical Potential Energy

* The idea of relating work done on a charge to potential energy is motivated by the problem of determining the change in potential energy of a charge after being moved a certain distance in an electric field.
* If we were to move a charge, q0, by an infinitesimally small distance, dl (from point “a” to point “b”, then the work done on that charge would be given by:
* Here, it should be noted that because the electrical field is **conservative**, the work done on an object will be **path independent**.
* **Change in potential energy**: Thus, given the law of conservation of energy, we know that if work is done, then energy is transferred to/from the object. The energy transferred causes a change in the potential energy of the object. We can therefore relate work and potential energy as thus:
* Why is the change in potential energy negative the work done? This is due to the definition of work: Force times displacement. When we move a particle from lower to higher electrical potential energy, this is in the opposite direction to the force exerted by the electric field. Hence, the work done would be negative (since displacement is in the opposite direction to force applied) while the change in potential energy is positive, hence deltaU = -W. However, such an answer depends on work having a direction.

## Voltage

Voltage refers to the potential difference between two points.

* Voltage, as a raw potential, is can be derived from the equation:
* We then set a reference point A, such that it is an infinite distance away from the point charge q, meaning that it has no potential. As such, we 1/ra approaches 0, so we are left with the equation:  
    
  a **distance** rb away from the point charge q.
* This can therefore be expressed in the equation  
    
   the potential at any point a distance r from a point charge.
* If we then place q at the center of a Cartesian coordinate system, we can derive the equation:
* Graph:
* Conductors: The field lines from a conductor will be perpendicular to its surface at all points. Hence, this implies that the entire region inside and on the surface of the conductor is equipotential.

## Voltage – Electrical Field

Here, Ua and Ub refers to the electrical potential energy of a charge q0, at points “a” and “b” respectively.

* From the above equation, the infinitesimal equation would then be:
* Thus, if we were to only move in the x axis, ie, the y and z position remains fixed, then we would have (through **partial differentiation)**:
* Likewise, we can extrapolate this result for the other dimensions:
* This therefore gives us the equation:   
    
  Or
* Here, the symbol is known as nabla or Del, and denotes:
* Therefore, represents the gradient of V. By extension, the electric field of an object indicates the negative rate of change of Voltage.

# Capacitance

A capacitator is any pair of conductors – one with a charge of n+Q and the other with a charge of n(–Q), separated by some distance. The capacitance, C, is defined to be

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Revolutions in Physics

# Radiation

## Stefan’s Law

* However, the problem with the Stefan-Boltzmann law was that it predicted an ultraviolet catastrophe, whereby all objects would be emitting radiation of infinite intensity.
* The ultraviolet catastrophe is resolved by Planck’s probabilistic interpretation of radiation, such that energy from radiation is quantised in packets of energy known as quanta.
* The probability of an emitter being in a certain energy level then decreases as we approach greater frequencies.

## Wavelength of Maximum Intensity

Where lambda represents the wavelength associated with the maximum intensity, and a is a constant whose value is 2.9\*10^-3.

* On the other hand, the median wavelength is given by:
* The above equation for median wavelength makes sense because the median wavelength is associated with the median intensity, which has less energy -> greater wavelength for a given temperature.

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# Photoelectric Effect

* The photoelectric effect was observed in an experiment where light illuminated one piece of metal, causing the emittance of **photoelectrons** (really just electrons, the prefix of –photo is because the electrons were emitted by photons).
* The photo electrons would then approach another piece of metal, which had a **negative voltage** to repel the incoming electrons.
* The negative voltage was then adjusted until all the electrons were repelled. This voltage is proportional to the maximum kinetic energy of the photo electrons.
* The key observations from the experiment were:
  + There was no time delay between the incidence of the light and emittance of photoelectrons.
  + Below a certain cut-off frequency, no photoelectrons would be emitted regardless of how high the frequency was.
  + The maximum kinetic energy of the photoelectrons (and therefore the maximum voltage) was only dependent on the frequency of the light.
  + The voltage difference is above a certain negative voltage value Vs.
* This indicated, as Planck suggested, that light could in fact be interpreted as particles of energy hf, and thus, the light must be above a certain frequency in order to overcome the work function of the metal (ie, the energy needed to liberate the electrons) and therefore cause emission.
* The relationship can be described as follows:  
    
  work function of the metal, and KE max represents the maximum kinetic energy of the photoelectron.
* **Note:** In the above equation, KE max can be measured (in eV) through the voltage required to repel **all** photoelectrons. This voltage is usually **negative**. However, if the voltage of the opposing metal surface is **positive**, this means that the electrons will be attracted towards the metal surface, and thus, KE max will be given by  
    
  voltage of the positively charged opposing metal surface.

# Momentum and Energy

## Bohr Atom

# Heisenberg’s Uncertainty Principle

* Essentially, Heisenberg’s Uncertainty Principle arises from the fact that due to the sensitive nature of microscopic objects, attempts to observe processes or objects will result in inaccuracies, meaning that there is a tension between accuracy in measurements of position, Deltax, and accuracy in measurements of momentum, Deltap.